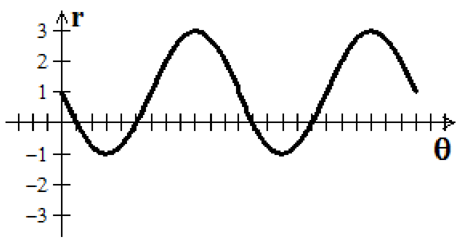
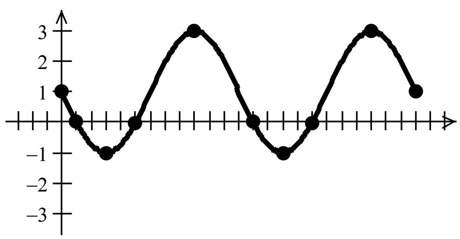
How to draw a rough sketch of the polar curve $r = f(\theta), \quad \theta \in [0, 2\pi]$ quickly while plotting only a few points when you cannot convert the polar equation into a rectangular equation with a recognizable graph using $r = 1 - 2\sin 2\theta$ as an example

NOTE: This technique only gives a complete graph if the period of $f(\theta)$ is $\frac{2\pi}{n}$ where n is an integer.

1. Sketch the graph of $r = f(\theta)$ for $\theta \in [0, 2\pi]$ on the Cartesian plane, with θ on the horizontal axis, and r on the vertical axis.



2. Determine the $\theta -$ values corresponding to the local maxima and minima of r, and to $r = 0$. Draw a number line for $\theta \in [0, 2\pi]$, and label those $\theta -$ values on it.



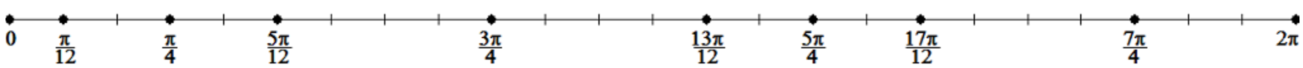
$r = 0:$

$1 - 2\sin 2\theta = 0 \quad \Rightarrow \quad \sin 2\theta = \frac{1}{2}$

$0 \le \theta \le 2\pi \quad \Rightarrow \quad 0 \le 2\theta \le 4\pi$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \quad \Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

The period of $\sin 2\theta$ is $\frac{2\pi}{2} = \pi$, and the extrema of sine occur at $\frac{1}{4}$ and $\frac{3}{4}$ of a period.
(The vertical shift, reflection and stretching do not change the $\theta -$ values where the extrema occur.)
So, the extrema occur at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ (in the first period) , $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$ and $\frac{3\pi}{4} + \pi = \frac{7\pi}{4}$ (in the second period).

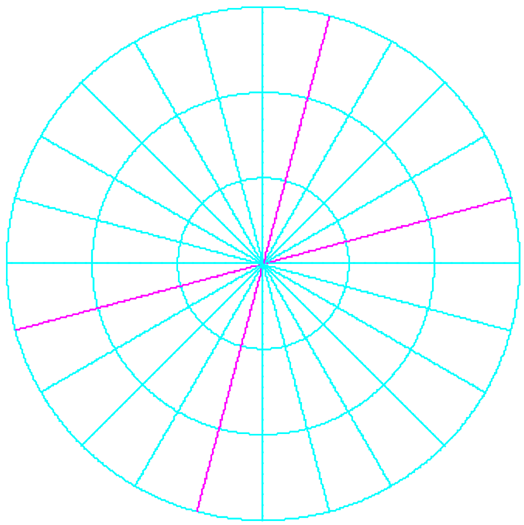


3. Between each successive pair of $\theta -$ values on the number line:
if $|r|$ is increasing (ie. the graph is moving away from the horizontal axis),
the polar graph will spiral counterclockwise out from the pole
if $|r|$ is decreasing (ie. the graph is moving towards the horizontal axis),
the polar graph will spiral counterclockwise in to the pole

From	$\theta = 0$	to	$\theta = \frac{\pi}{12}$,	$\lvert r \rvert$ is decreasing,	so the polar graph is spiraling in to the pole
From	$\theta = \frac{\pi}{12}$	to	$\theta = \frac{\pi}{4}$,	$\lvert r \rvert$ is increasing,	so the polar graph is spiraling out from the pole
From	$\theta = \frac{\pi}{4}$	to	$\theta = \frac{5\pi}{12}$,	$\lvert r \rvert$ is decreasing,	so the polar graph is spiraling in to the pole
From	$\theta = \frac{5\pi}{12}$	to	$\theta = \frac{3\pi}{4}$,	$\lvert r \rvert$ is increasing,	so the polar graph is spiraling out from the pole
From	$\theta = \frac{3\pi}{4}$	to	$\theta = \frac{13\pi}{12}$,	$\lvert r \rvert$ is decreasing,	so the polar graph is spiraling in to the pole
From	$\theta = \frac{13\pi}{12}$	to	$\theta = \frac{5\pi}{4}$,	$\lvert r \rvert$ is increasing,	so the polar graph is spiraling out from the pole
From	$\theta = \frac{5\pi}{4}$	to	$\theta = \frac{17\pi}{12}$,	$\lvert r \rvert$ is decreasing,	so the polar graph is spiraling in to the pole
From	$\theta = \frac{17\pi}{12}$	to	$\theta = \frac{7\pi}{4}$,	$\lvert r \rvert$ is increasing,	so the polar graph is spiraling out from the pole
From	$\theta = \frac{7\pi}{4}$	to	$\theta = 2\pi$,	$\lvert r \rvert$ is decreasing,	so the polar graph is spiraling in to the pole

4. Start drawing the polar graph by drawing dotted lines through the pole corresponding to the $\theta-$ values where $r = 0$.
At these values of θ, the polar graph will go through the pole.
As the polar graph enters and leaves the pole, the polar graph will be tangent to those dotted lines.

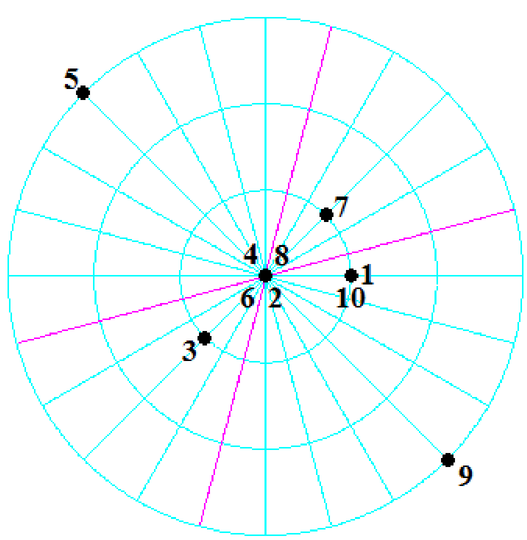
NOTE: Tangent lines drawn as pink lines below



5. Plot the polar points corresponding to the $\theta-$ values on the number line in step 2.
(Be careful which quadrant those points are in.)

[1]	$\theta = 0$	$\Rightarrow$	$r = 1 - 2\sin 2(0) = 1 - 2\sin 0 = 1$	θ and point are both on positive polar axis
[2]	$\theta = \frac{\pi}{12}$	$\Rightarrow$	$r = 0$	pole
[3]	$\theta = \frac{\pi}{4}$	$\Rightarrow$	$r = 1 - 2\sin 2(\frac{\pi}{4}) = 1 - 2\sin \frac{\pi}{2} = -1$	θ is in Q_1, but point is in Q_3 since $r < 0$
[4]	$\theta = \frac{5\pi}{12}$	$\Rightarrow$	$r = 0$	pole
[5]	$\theta = \frac{3\pi}{4}$	$\Rightarrow$	$r = 1 - 2\sin 2(\frac{3\pi}{4}) = 1 - 2\sin \frac{3\pi}{2} = 3$	θ and point are both in Q_2
[6]	$\theta = \frac{13\pi}{12}$	$\Rightarrow$	$r = 0$	pole
[7]	$\theta = \frac{5\pi}{4}$	$\Rightarrow$	$r = 1 - 2\sin 2(\frac{5\pi}{4}) = 1 - 2\sin \frac{5\pi}{2} = -1$	θ is in Q_3, but point is in Q_1 since $r < 0$
[8]	$\theta = \frac{17\pi}{12}$	$\Rightarrow$	$r = 0$	pole
[9]	$\theta = \frac{7\pi}{4}$	$\Rightarrow$	$r = 1 - 2\sin 2(\frac{7\pi}{4}) = 1 - 2\sin \frac{7\pi}{2} = 3$	θ and point are both in Q_4
[10]	$\theta = 2\pi$	$\Rightarrow$	$r = 1 - 2\sin 2(0) = 1 - 2\sin 0 = 1$	θ and point are both on positive polar axis

NOTE: Numbering indicates order of points along curve



6. Connect the points in step 5 by spiraling counterclockwise as stated in step 3.
   Be mindful of the tangent lines in step 4.

NOTE: Pink tangent lines are not part of the polar graph
Green sections of curve are spiraling out from the pole (based on step 3)
Red sections of curve are spiraling in to the pole (based on step 3)